LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2023**

PST1MC03 – STATISTICAL MATHEMATICS

Date: 03-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION A			
Answer ALL the questions			
1	**Define the following.** **(5 x 1 = 5)**		
a)	Strictly monotonically increasing sequence	K1	CO1
b)	Extreme value	K1	CO1
c)	Riemann Integral	K1	CO1
d)	Linear Span	K1	CO1
e)	Minimal polynomial	K1	CO1
2	**Fill in the blanks.** **(5 x 1 = 5)**		
a)	The nature of an infinite series remains unaltered if a finite number of terms are added or_______________.	K2	CO1
b)	$\lim\limits_{x\to 0}\frac{a^x - b^x}{x}$ is equal to ________________.	K2	CO1
c)	Let $f(x) = k$ (= constant) on [a,b] and g be monotonically, non-decreasing on [a,b]. Then $\int_a^b f\ dg =$ _________.	K2	CO1
d)	Any infinite set of vectors of V is linearly independent if its every finite subset is linearly __________	K2	CO1
e)	The characteristic roots of a skew-hermitian matrix are ________________	K2	CO1
SECTION B			
	Answer any THREE of the following questions. **(3 x 10 = 30)**		
3	Prove that the sequence $\{a_n\}$ defined by $a_n = \left(1+\frac{1}{n}\right)^n$ is convergent.	K3	CO2
4	Discuss the points of discontinuity of the function defined on [0,1] as follows: $f(x) = \begin{cases} 0 & if\ x = 0 \\ \frac{1}{2} - x & if\ 0 < x < \frac{1}{2} \\ \frac{1}{2} & if\ x = \frac{1}{2} \\ \frac{3}{2} - x & if\ \frac{1}{2} < x < 1 \\ 1 & if\ x = 1 \end{cases}$. Also, examine the kinds of discontinuities.	K3	CO2
5	Show that the vectors $\alpha_1 = (1,2,1),\ \alpha_2 = (2,1,0),\ \alpha_3 = (1,-1,\ 2)$ form a basis for R^3. Express each of the standard basis vectors as a linear combination of $\alpha_1,\ \alpha_2,\ \alpha_3$.	K3	CO2
6	If $f \in R[a,b]$, then prove that (i) $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)\ if\ b \ge a$ (ii) $m(b-a) \ge \int_a^b f(x)\,dx \ge M(b-a)\ if\ b \le a$; where m and M are the infimum and supremum of f on [a,b].	K3	CO2

7	Determine the characteristic roots and the corresponding characteristic vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and also show that the matrix satisfies Cayley-Hamilton theorem.	K3	CO2
	SECTION C		
	Answer any TWO of the following questions. (2 x 12.5 = 25)		
8	If $\{a_n\}$ is a sequence of positive real numbers such that $a_n = \frac{1}{2}(a_{n-1} + a_{n-2}) \forall n \geq 3$, then prove that $\{a_n\}$ converges to $\frac{1}{3}(a_1 + 2a_2)$.	K4	CO3
9	Evaluate the following limits (i) $\underset{x \to 0}{Lt}\left(\cot^2 x - \frac{1}{x^2}\right)$ (ii) $\underset{x \to 0}{Lt}\left(\frac{(1+x)^{1/x} - e}{x}\right)$	K4	CO3
10	If S, T are two subsets of a vector space V, then prove that (i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ (ii) $L(SUT) = L(S) + L(T)$ (iii) $L[L(S)] = L(S)$.	K4	CO3
11	(i)Apply Gram-Scmidt orthogonalization process to the vectors $\beta_1 = (3,0,4)$, $\beta_2 = (-1,0,7)$, $\beta_3 = (2,9,11)$ to obtain an orthonormal basis $\{\alpha_1, \alpha_2, \alpha_3\}$ for R^3 with standard inner product. (ii).Which of the following functions T from R^2 into R^2 are linear transformation? (a). $T(x_1, x_2) = (x_1-x_2, 0)$, (b). $T(x_1, x_2) = (\sin x_1, x_2)$.	K4	CO3

	SECTION D		
	Answer any ONE of the following questions (1 x 15 = 15)		
12	(i) Using Cauchy's condensation test, test the convergence of the series $u_n = \frac{1}{(n \log n)^p}$, $n \geq 2$, (7+8) (ii) Test the convergence of the series $\sum \frac{n!}{x(x+1)(x+2)...(x+n-1)}$	K5	CO4
13	Reduce the following quadratic form to real canonical form and find its rank and signature $x^2 + 4y^2 + 9z^2 + t^2 - 12yz + 6zx - 4xy - 2xt - 6zt$.	K5	CO4
	SECTION E		
	Answer any ONE of the following questions (1 x 20 = 20)		
14	Justify that every monotonic sequence either converges or diverges.	K6	CO5
15	(i).Test whether the following functions continuous and differentiable? (10) (a) $f(x) = 1 + x \text{ if } x < 2$ and $f(x) = 5 - x \text{ if } x \geq 2$ at the point $x = 2$. (b) $f(x) = 2 + x \text{ if } x \geq 0$ and $f(x) = 2 - x \text{ if } x < 0$ at the origin. (ii). Find the maximum and minimum values of the function $\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x,\ 0 \leq x \leq \pi$. (10)	K6	CO5

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